PROBLEM

Determine convergence or divergence of the following sequences and calculate limits where applicable:

$a)\ (-1)^{n}$

$b)\ (-1)^{n^2+n}$

$c)\ (n^2+n)\cdot(-1)^{n^2+n}$

$d)\ \dfrac{n+1}{xn^2+n}$ for different values of x

$e)\ (-1)^{n}\cdot\dfrac{n+1}{xn^2+n}$ for different values of x

SOLUTION

a) $(-1)^n$; the sequence oscillates between -1 and +1, thus divergent

b) $n^2+n=n(n+1)$ equals a product of two consecutive numbers, and thus always constitutes an even number. All numbers in the sequence $(-1)^{n^2+n}$ consequently are equal 1, thus the sequence is convergent.

c) $(-1)^{n^2+n}(n^2+n)=n^2+n$ according to b), strictly increasing and thus divergent.

d) $u_n=\dfrac{n+1}{xn^2+n}=\dfrac{1+1/n}{xn+1}$; $x=0$ yields $u_n=1+1/n\to 1$ when $n\to\infty$, convergent.

$x\neq 0$ yields $u_n=\dfrac{1+1/n}{xn+1}\to 0$ when $n\to\infty$, convergent.

e) $u_n=(-1)^n\cdot\dfrac{n+1}{xn^2+n}=(-1)^n\cdot\dfrac{1+1/n}{xn+1}$; $x=0\to u_n=(-1)^n(1+1/n)$ which oscillates, thus divergent. $x\neq 0\Rightarrow u_n\to 0$, thus convergent.

PROBLEM

Calculate the area defined by the closed loop

$x=t\cos t-t,\ y=t\sin t,\ \ 0\le t\le 2\pi$

SOLUTION

Loop area = $\displaystyle\int_{t_0}^{t_1}-y\frac{dx}{dt}\,dt=\int_{t_1}^{t_0}-x\frac{dy}{dt}\,dt=\frac{1}{2}\int_{t_0}^{t_1}(xy'-yx')\,dt,$

$x=t\cos t-t;\ x'=-t\sin t+\cos t-1;\ \ y=t\sin t;\ y'=t\cos t+\sin t\Rightarrow$

$$A=\frac{1}{2}\int_0^{2\pi}t^2(1-\cos t)dt=\frac{1}{2}\int_0^{2\pi}t^2dt-\frac{1}{2}\int_0^{2\pi}t^2\cos t\,dt=\frac{4\pi^3}{3}-\left[\frac{t^2}{2}\sin t\right]_0^{2\pi}+$$

$$+\int_0^{2\pi}t\sin t\,dt=\frac{4\pi^3}{3}-0+\left[-t\cos t\right]_0^{2\pi}+\int_0^{2\pi}\cos t\,dt=\frac{4\pi^3}{3}+0-2\pi+0=$$

$$=\frac{4\pi^3}{3}-2\pi$$